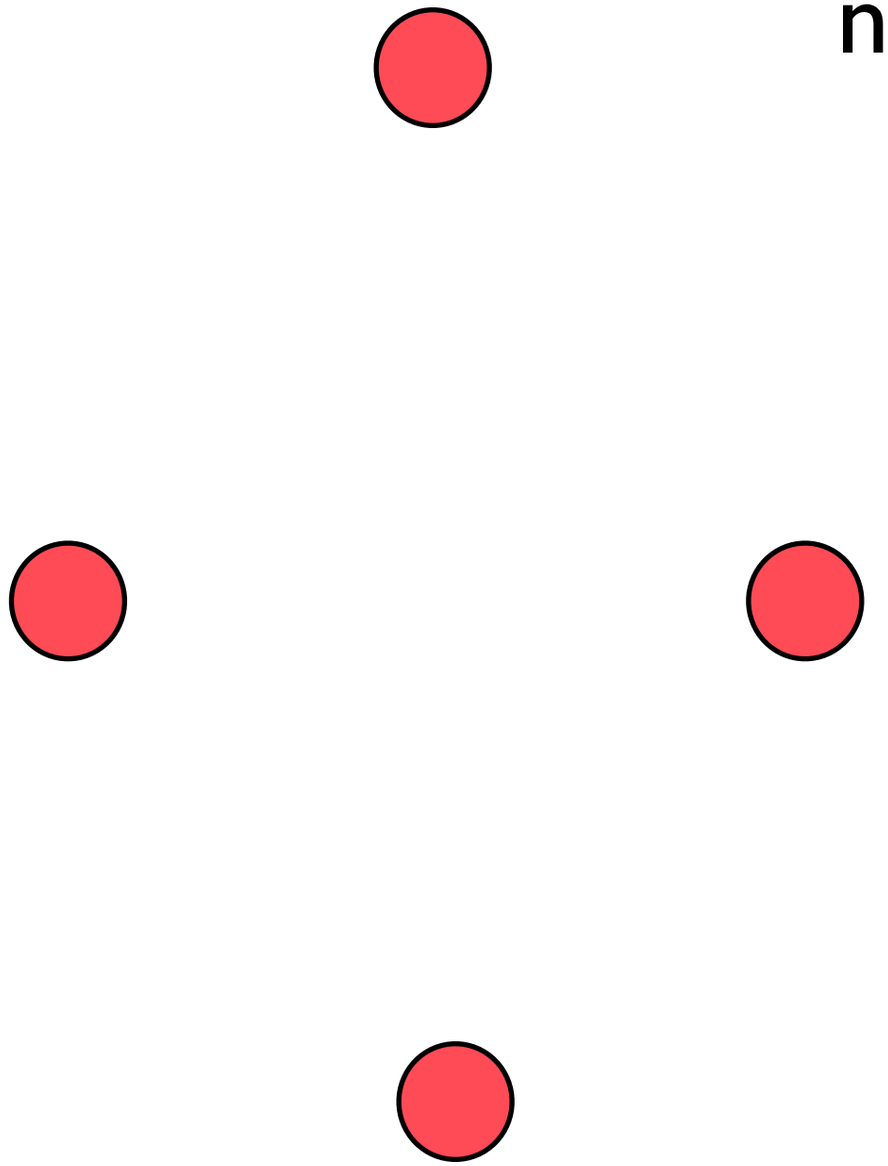
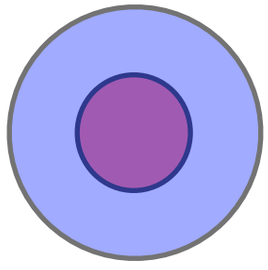
Studies of Combinatorial Extremal Problems Using High Performance Computing

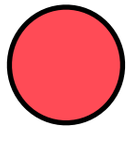
Klas Markström

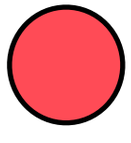
- Covering designs are used in testing and statistics
- The aim is to cover a set of objects as efficiently as possible
- We might want to cover either single objects or combinations of them
- In general, we have n objects and want to cover every t-tuple, using k objects at a time

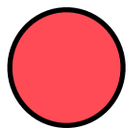
n=4, t=1, k=1

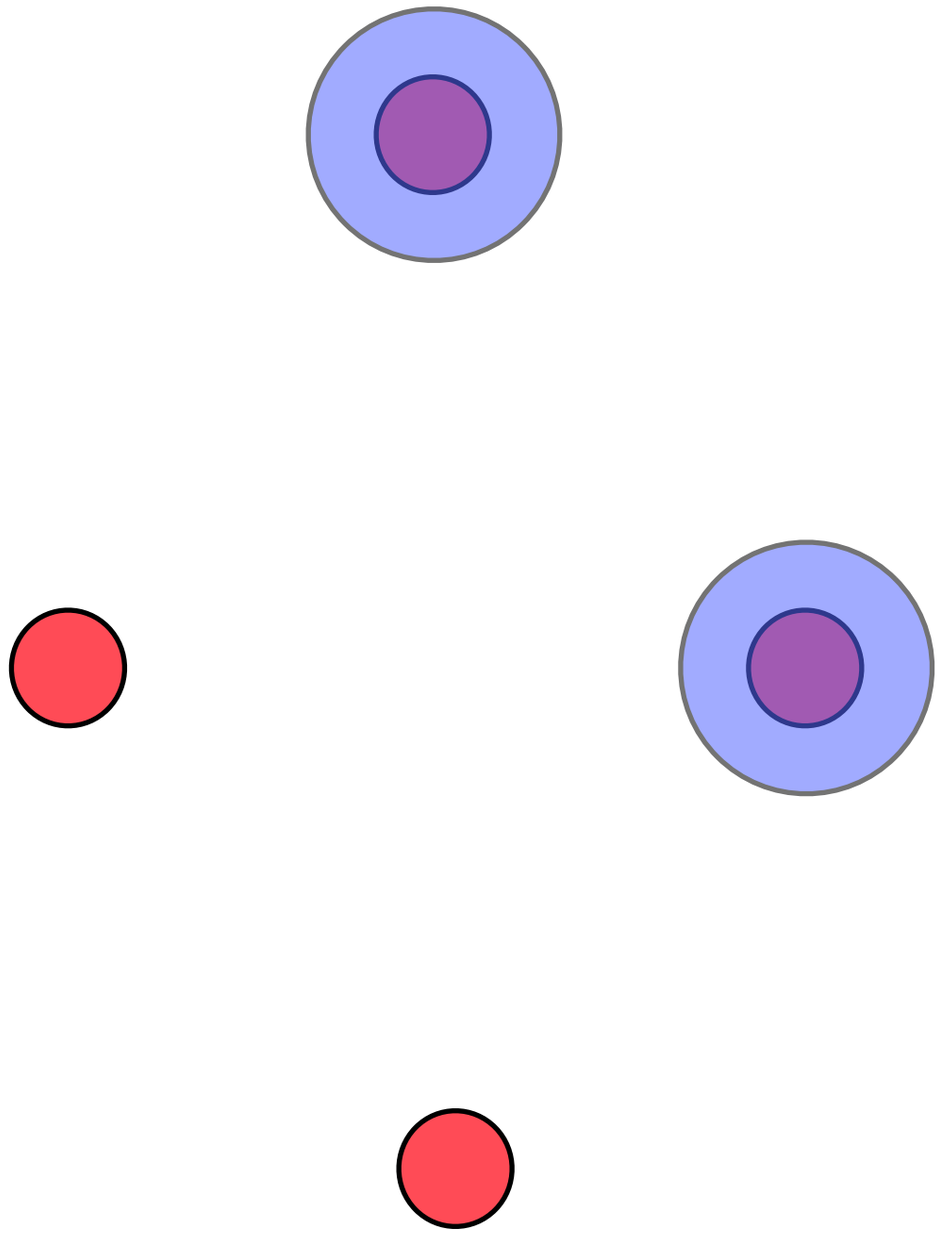


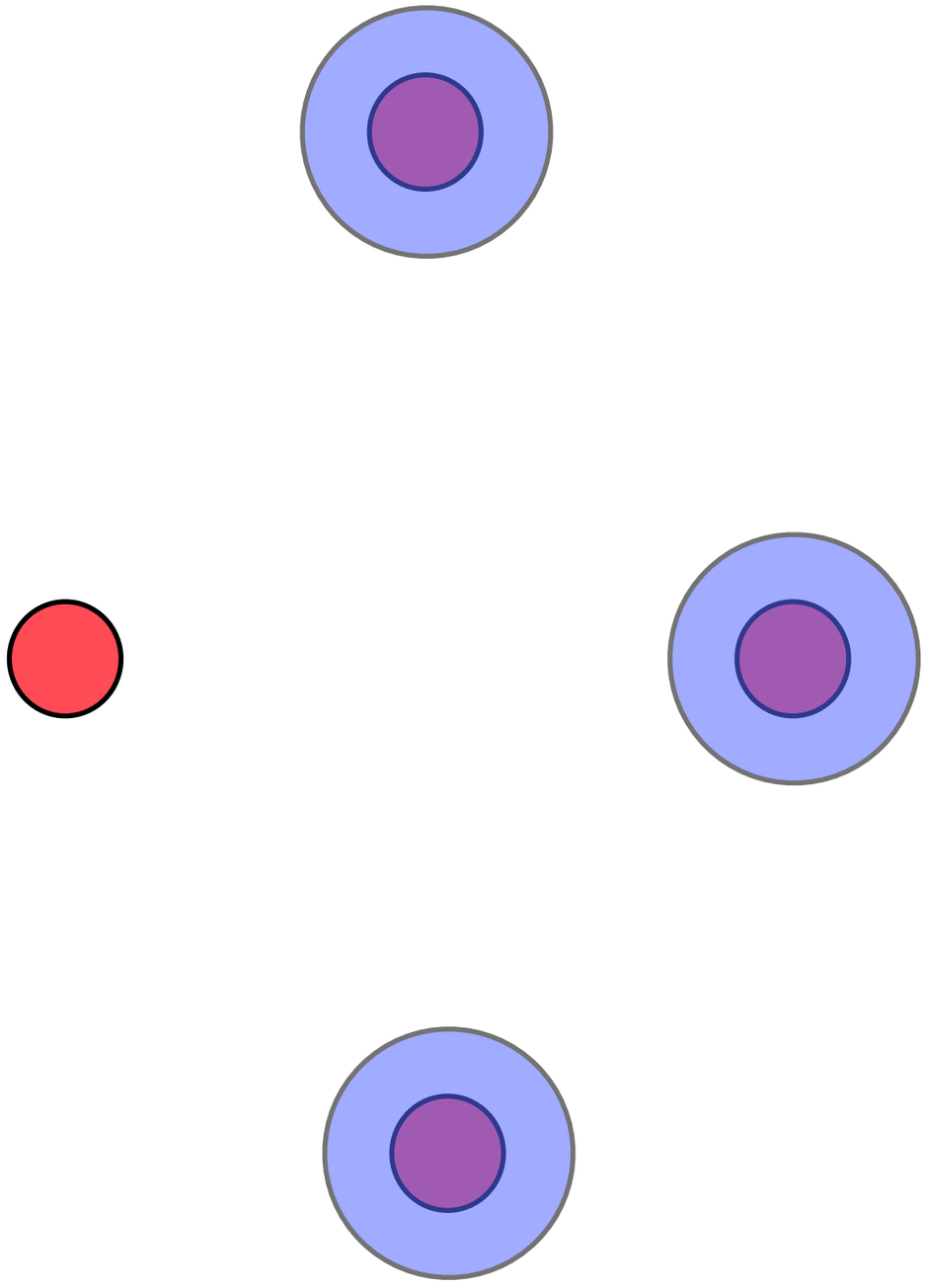


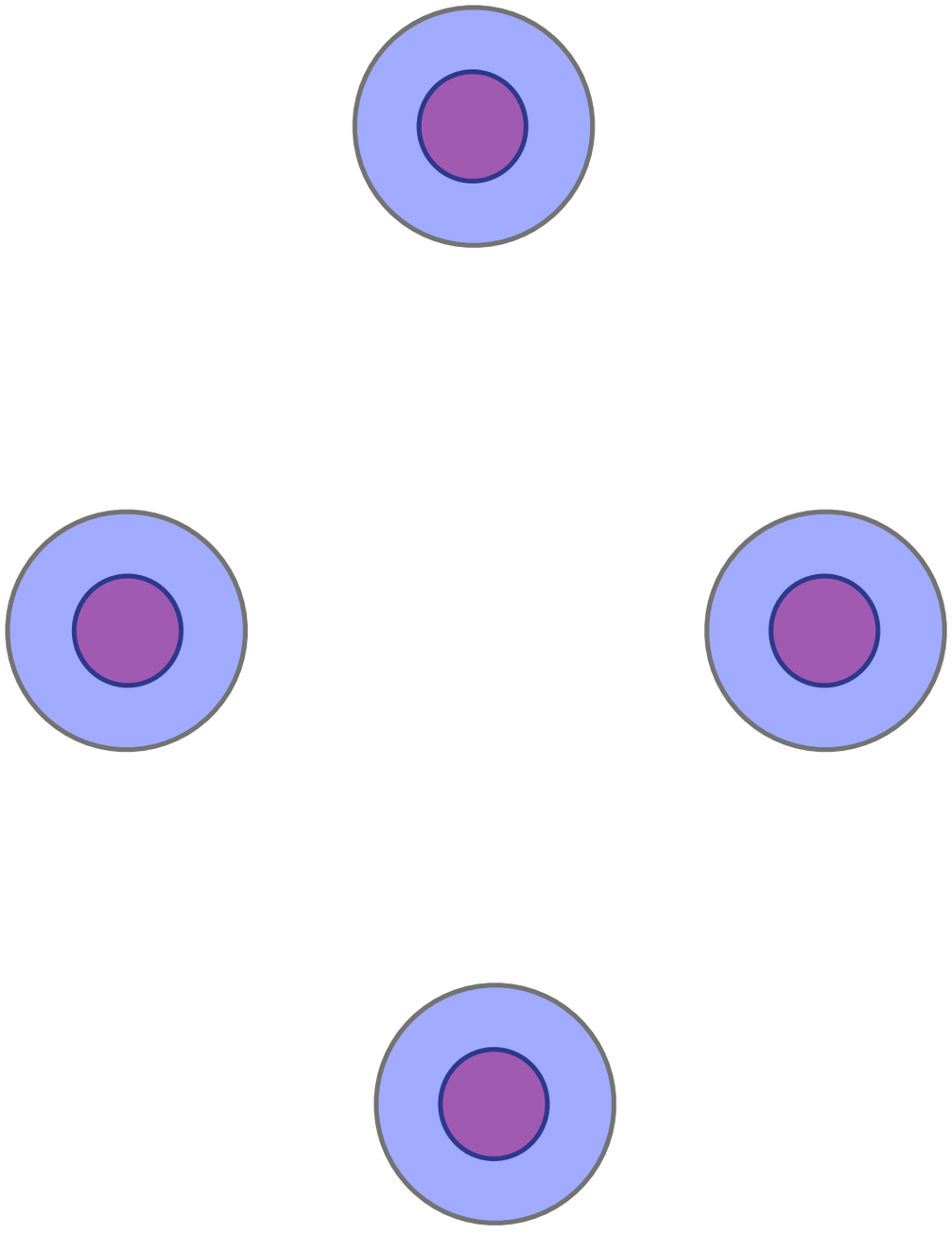




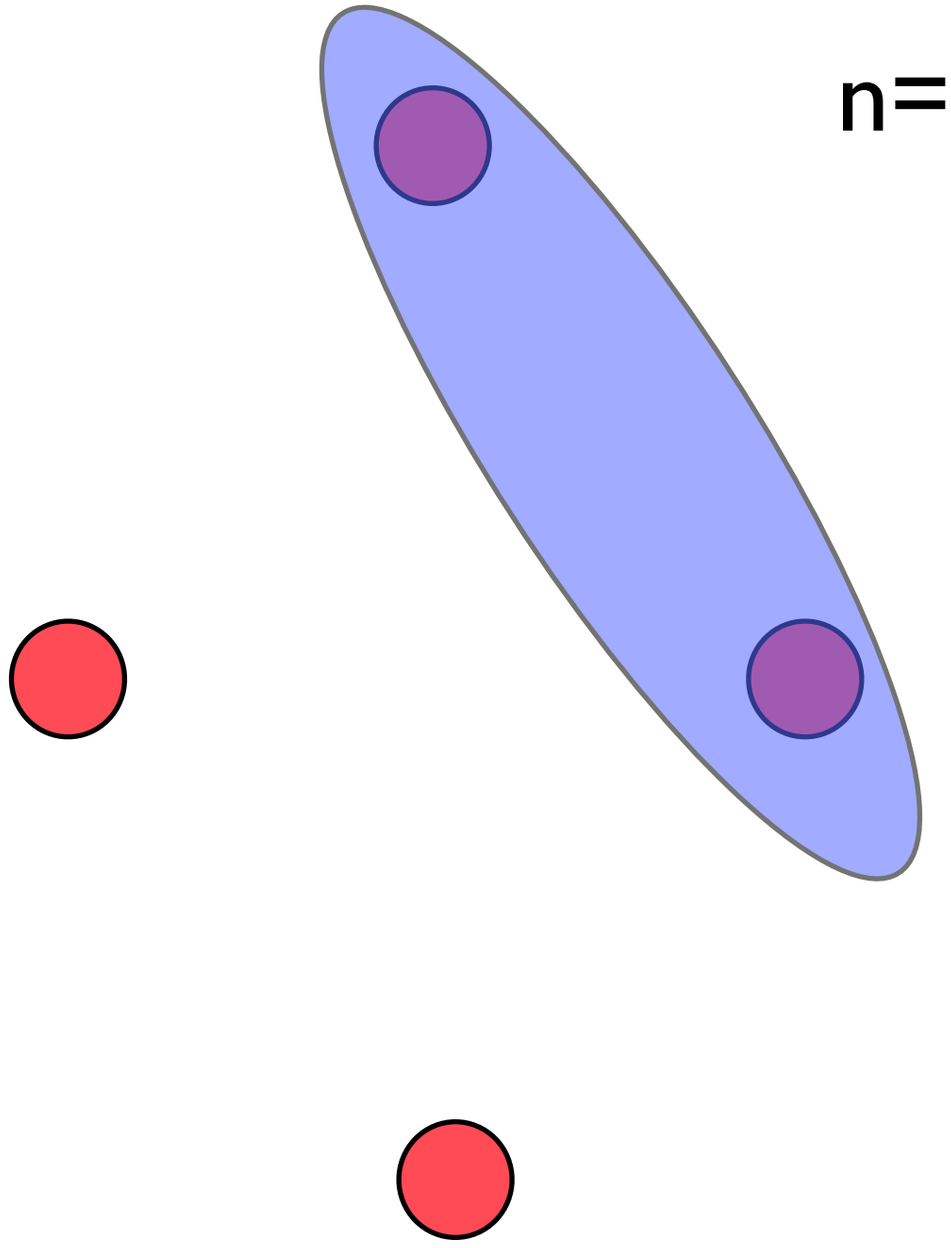


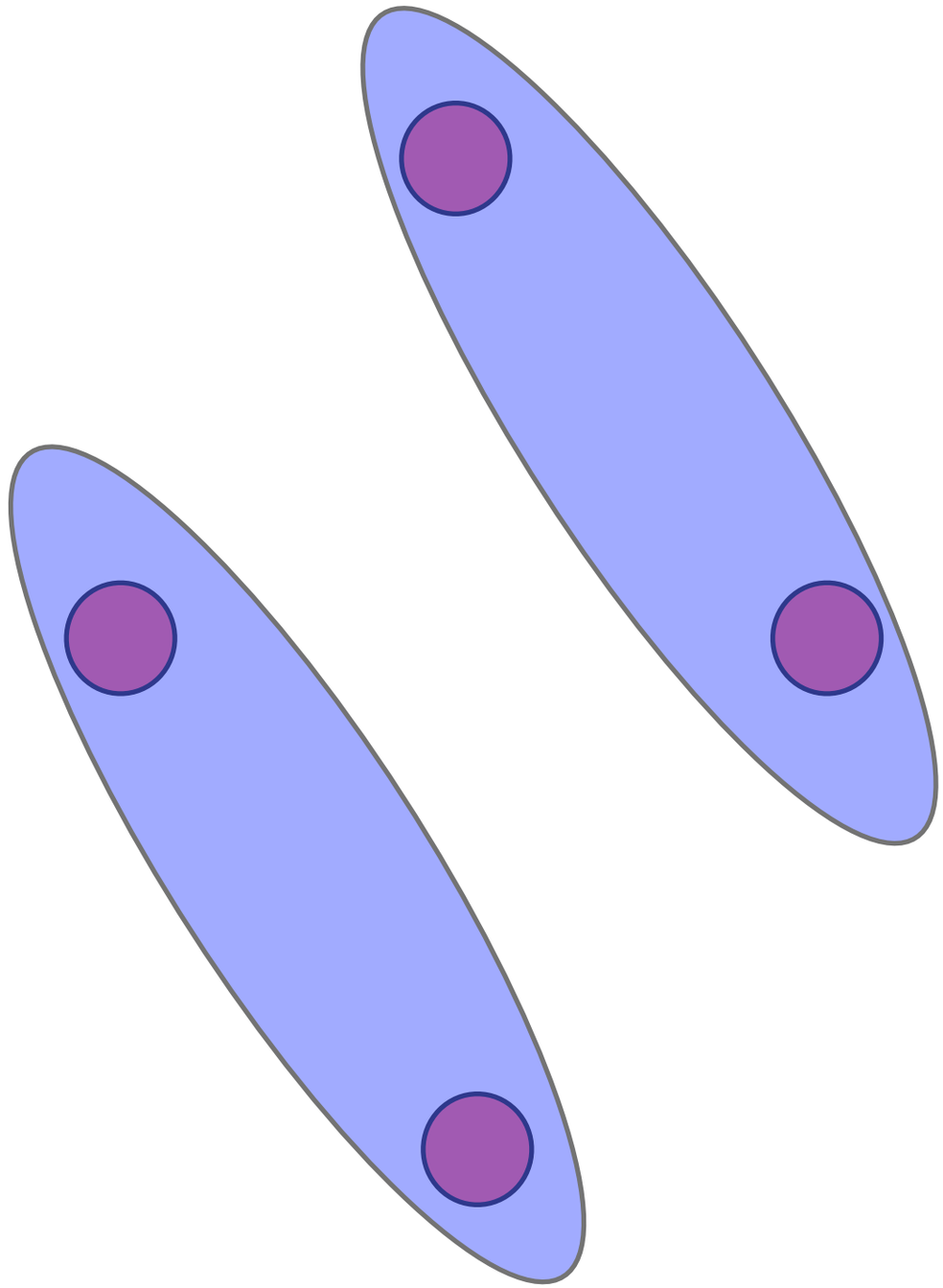




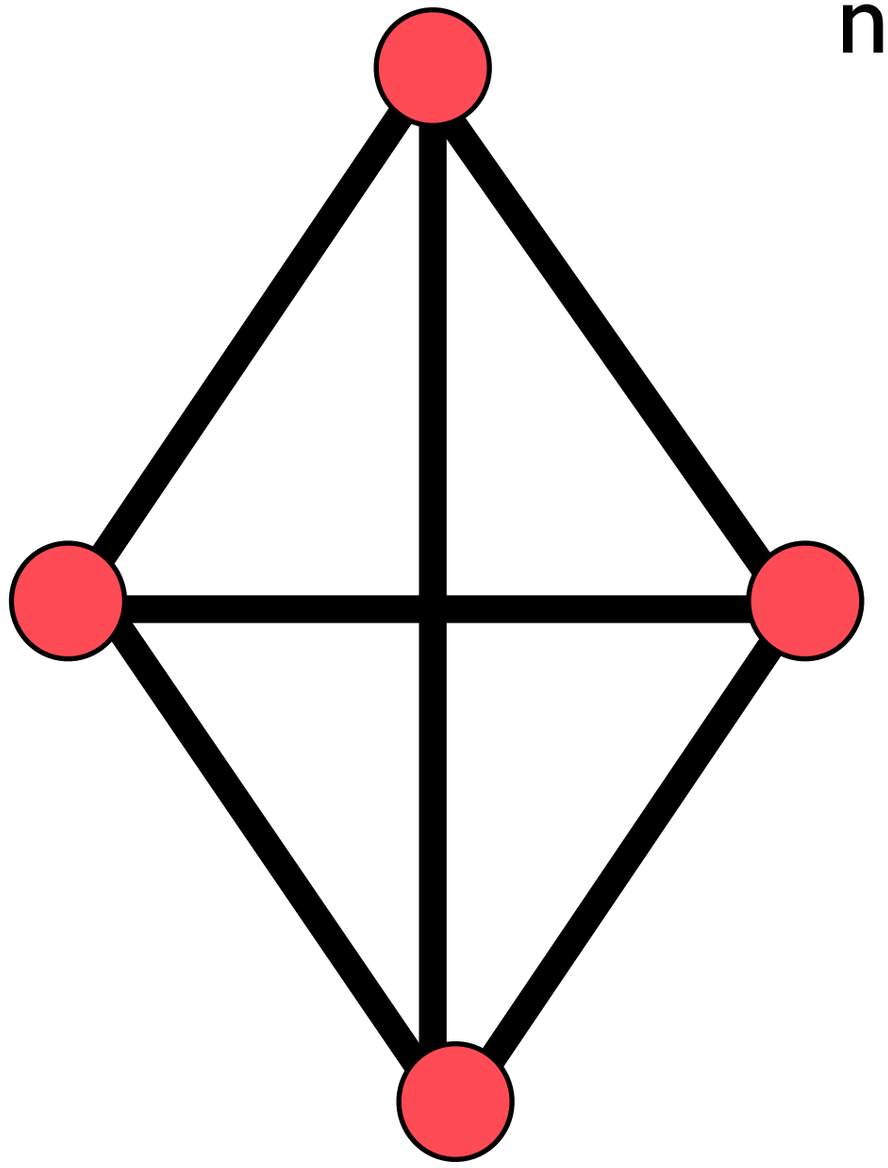


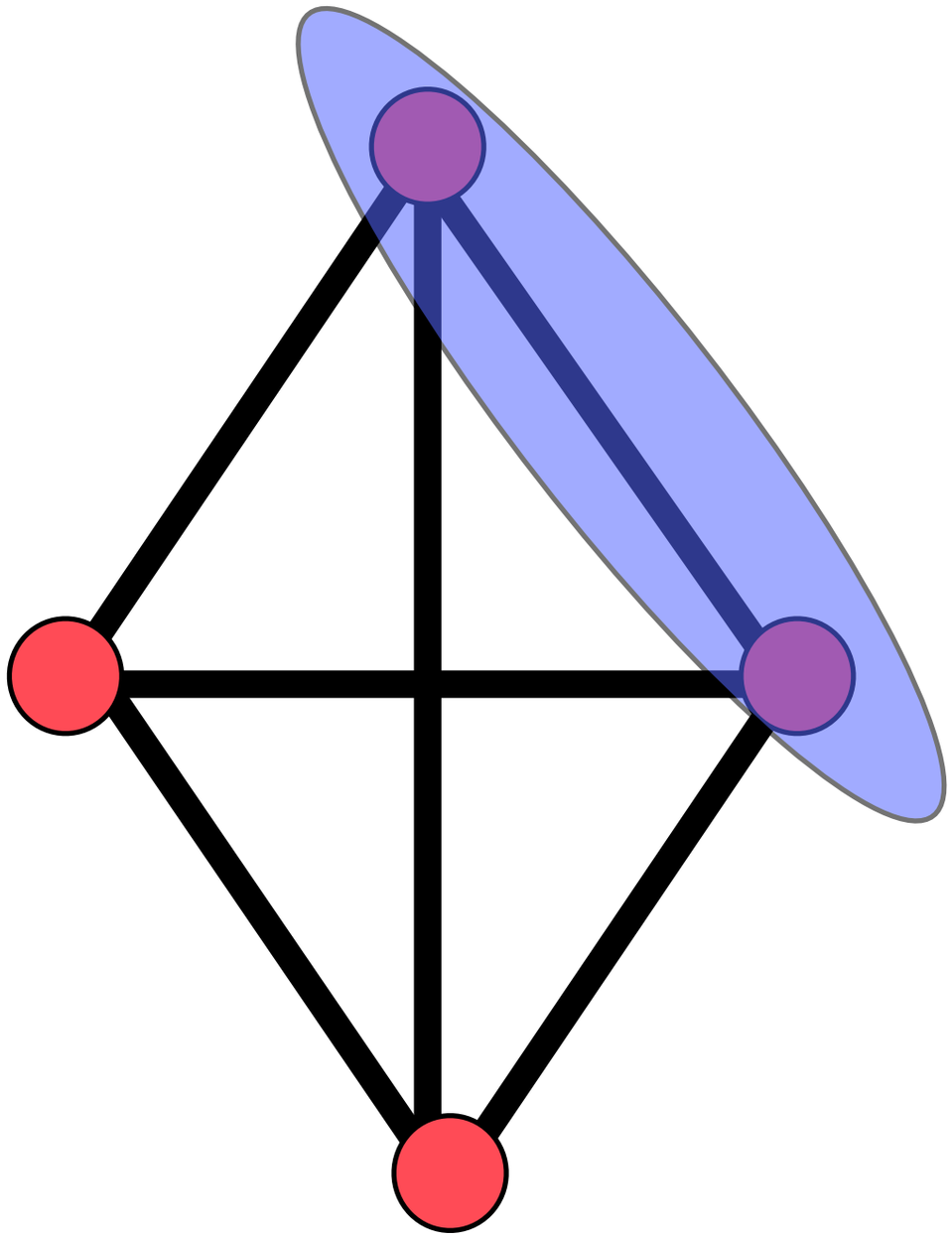
n=4, t=1, k=2

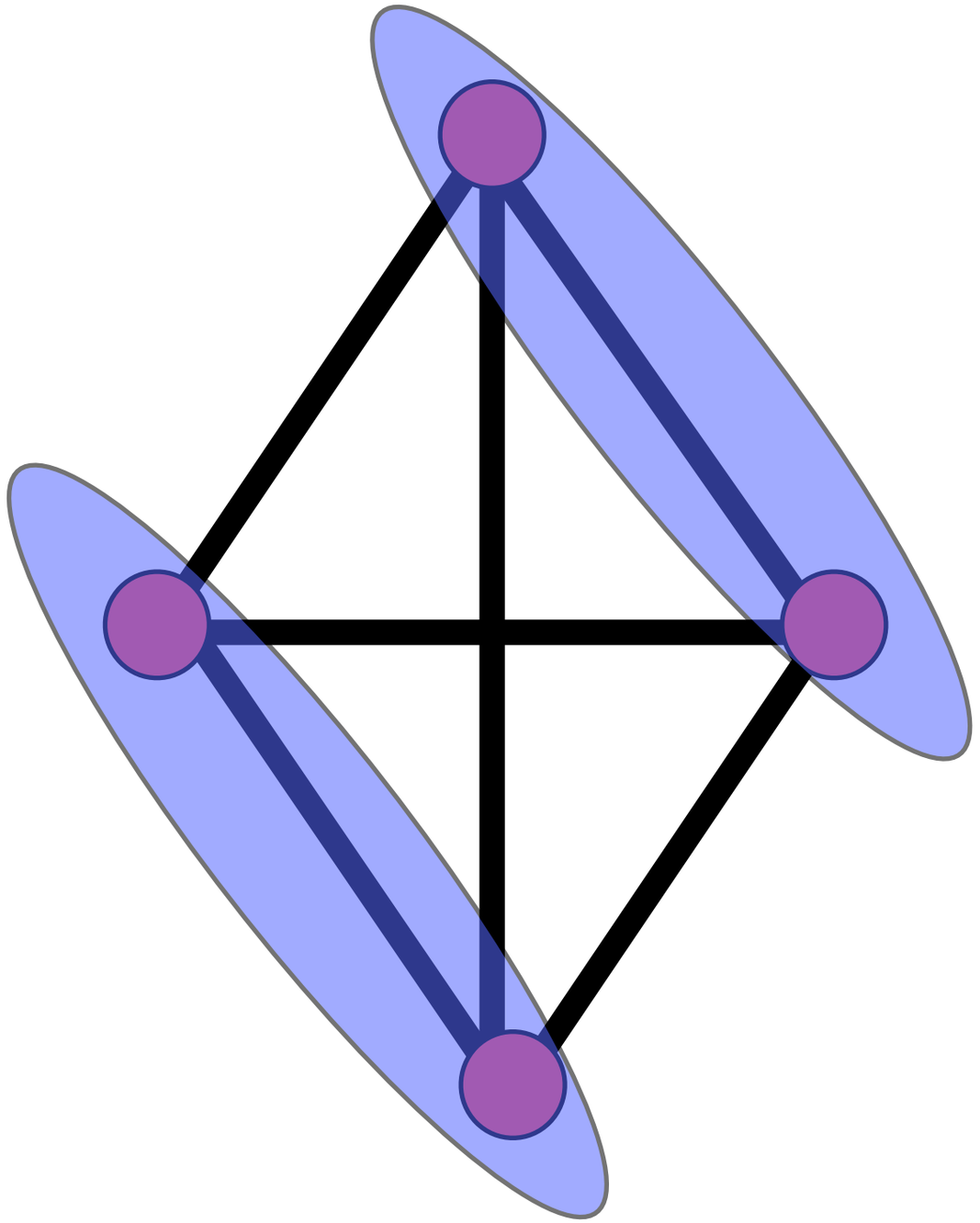


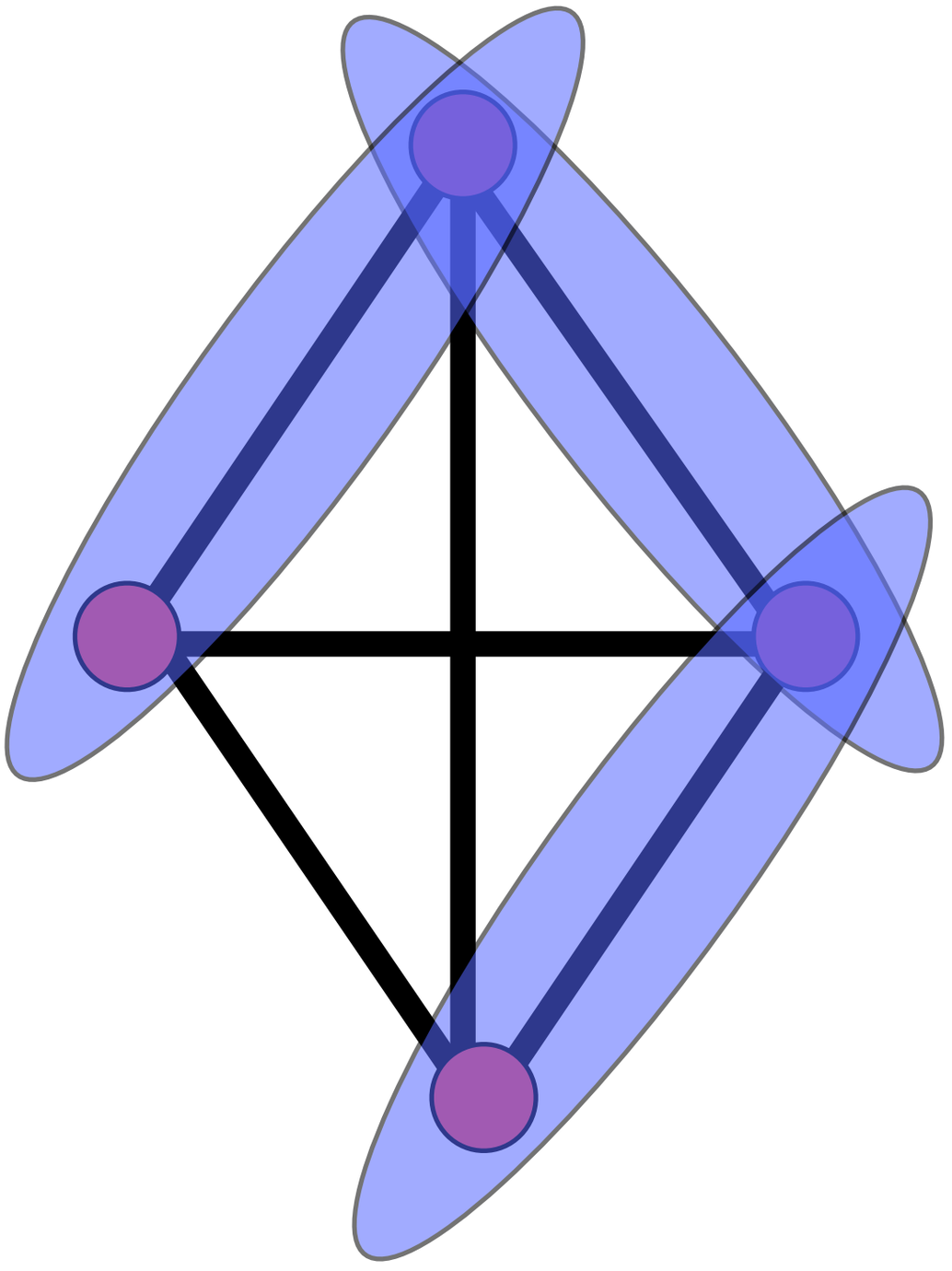


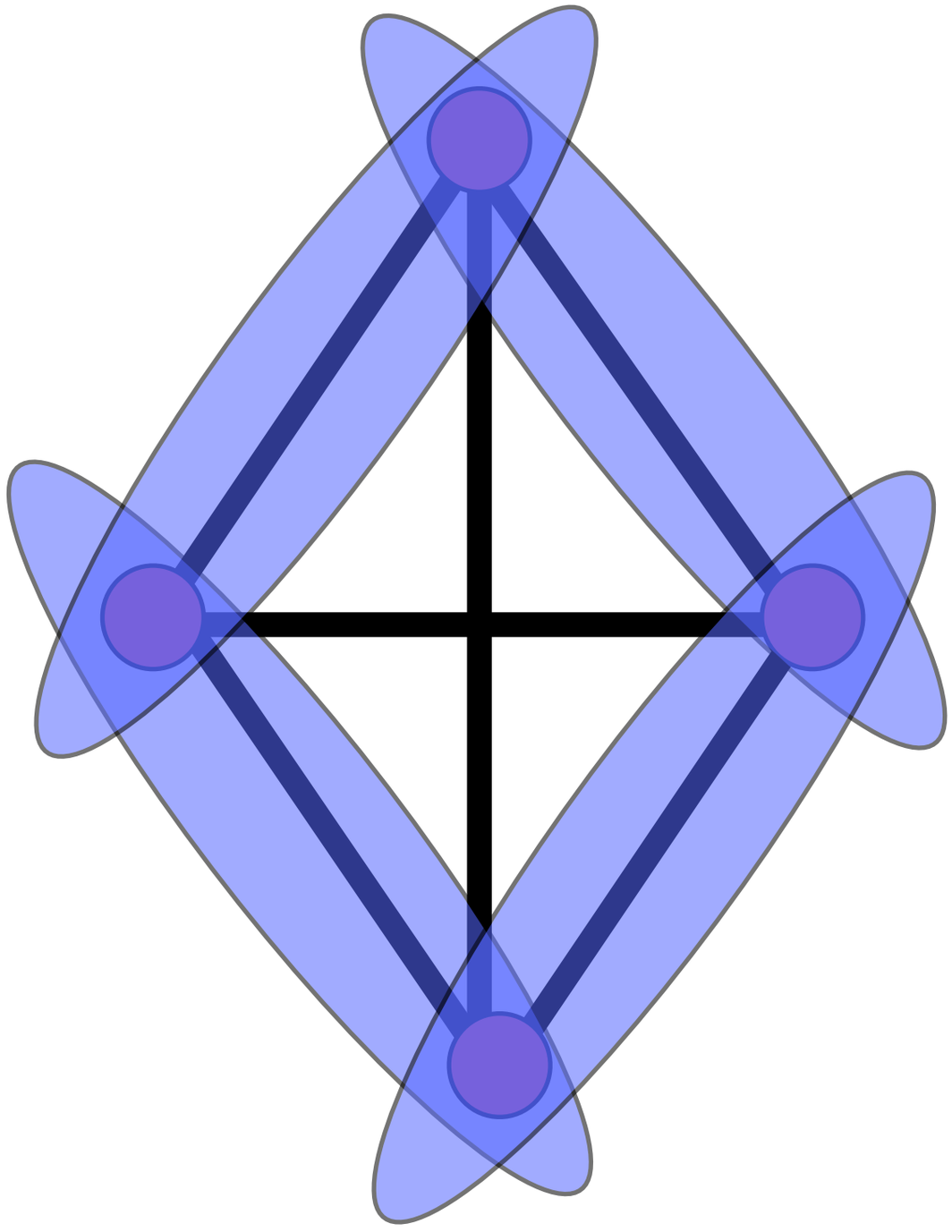
n=4, t=2, k=2

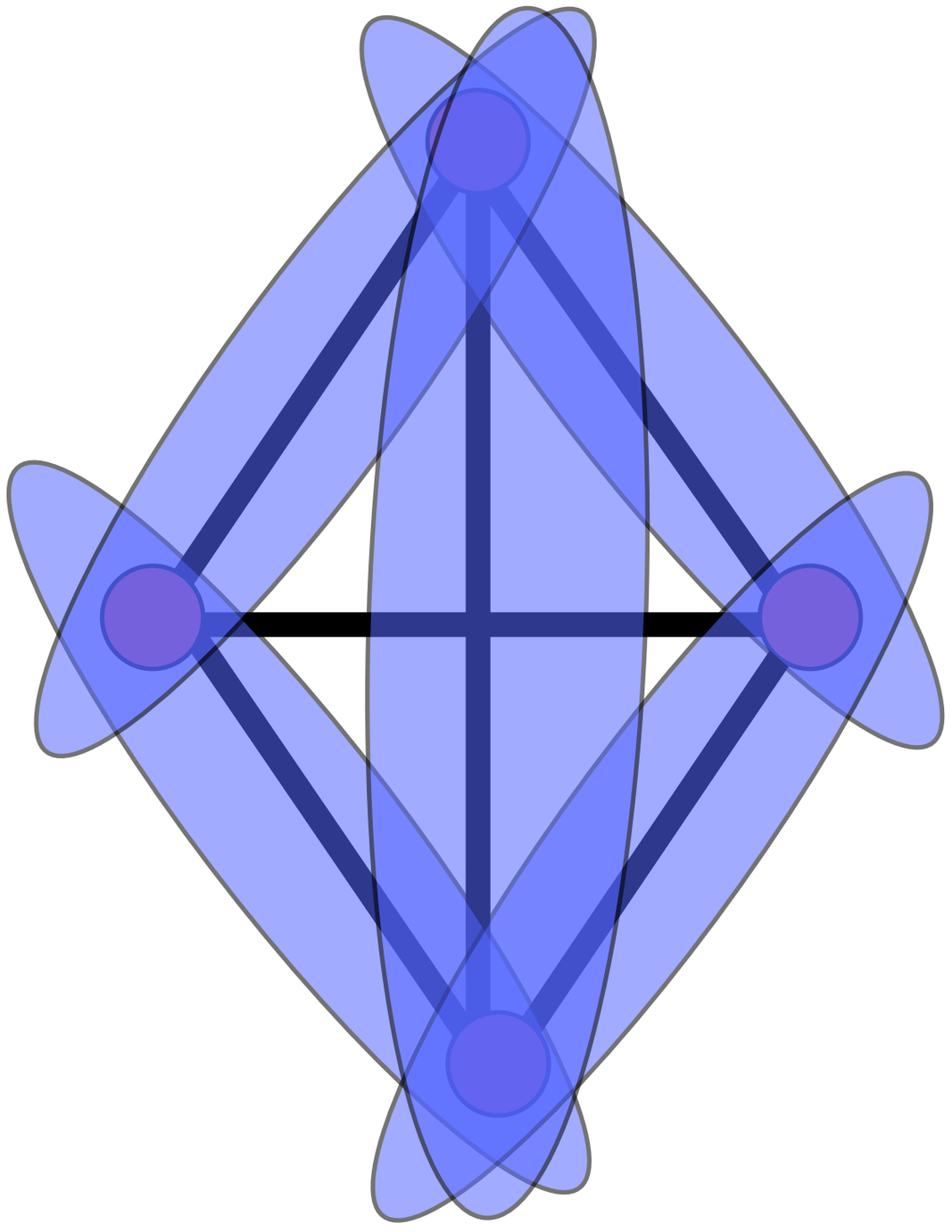


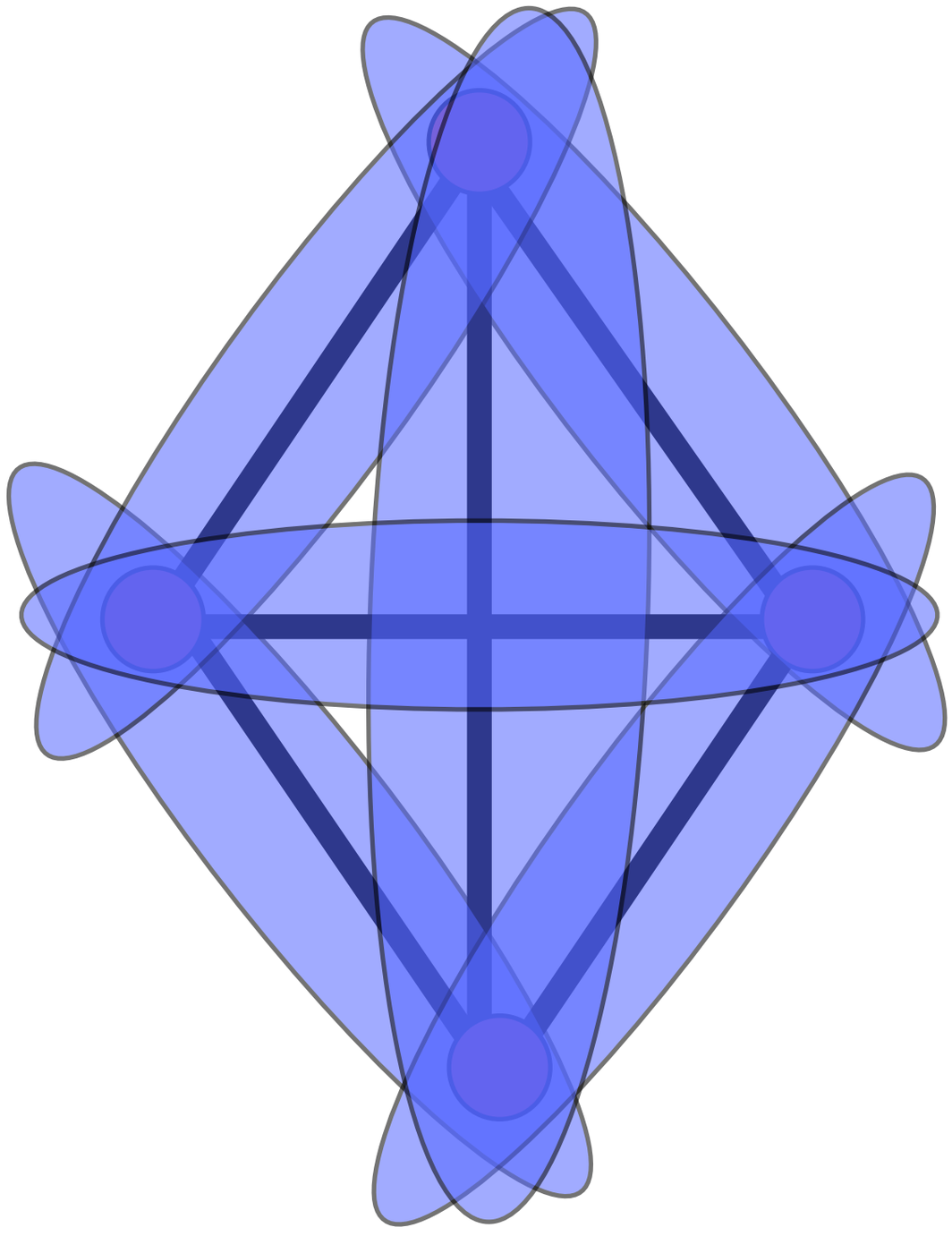


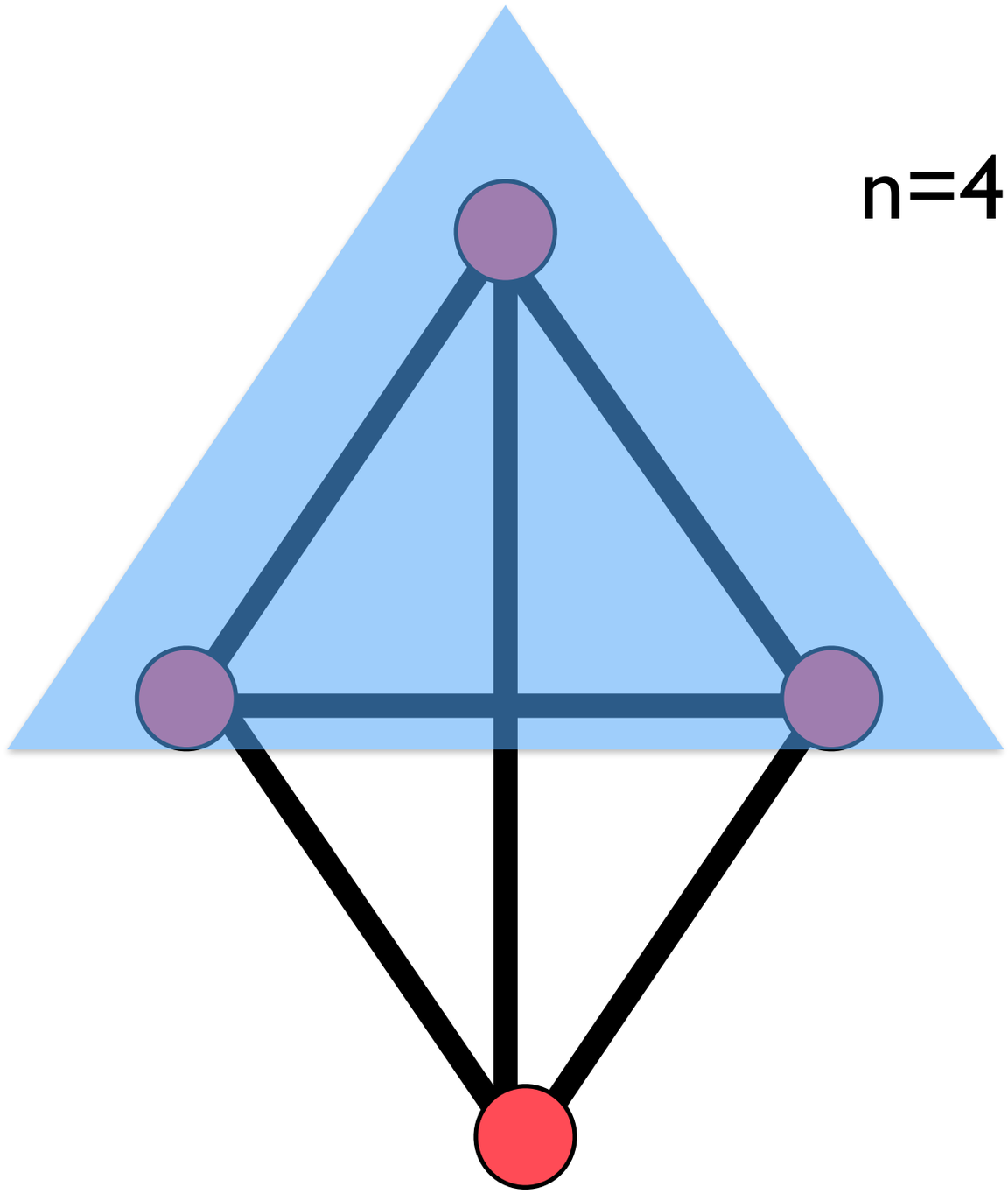




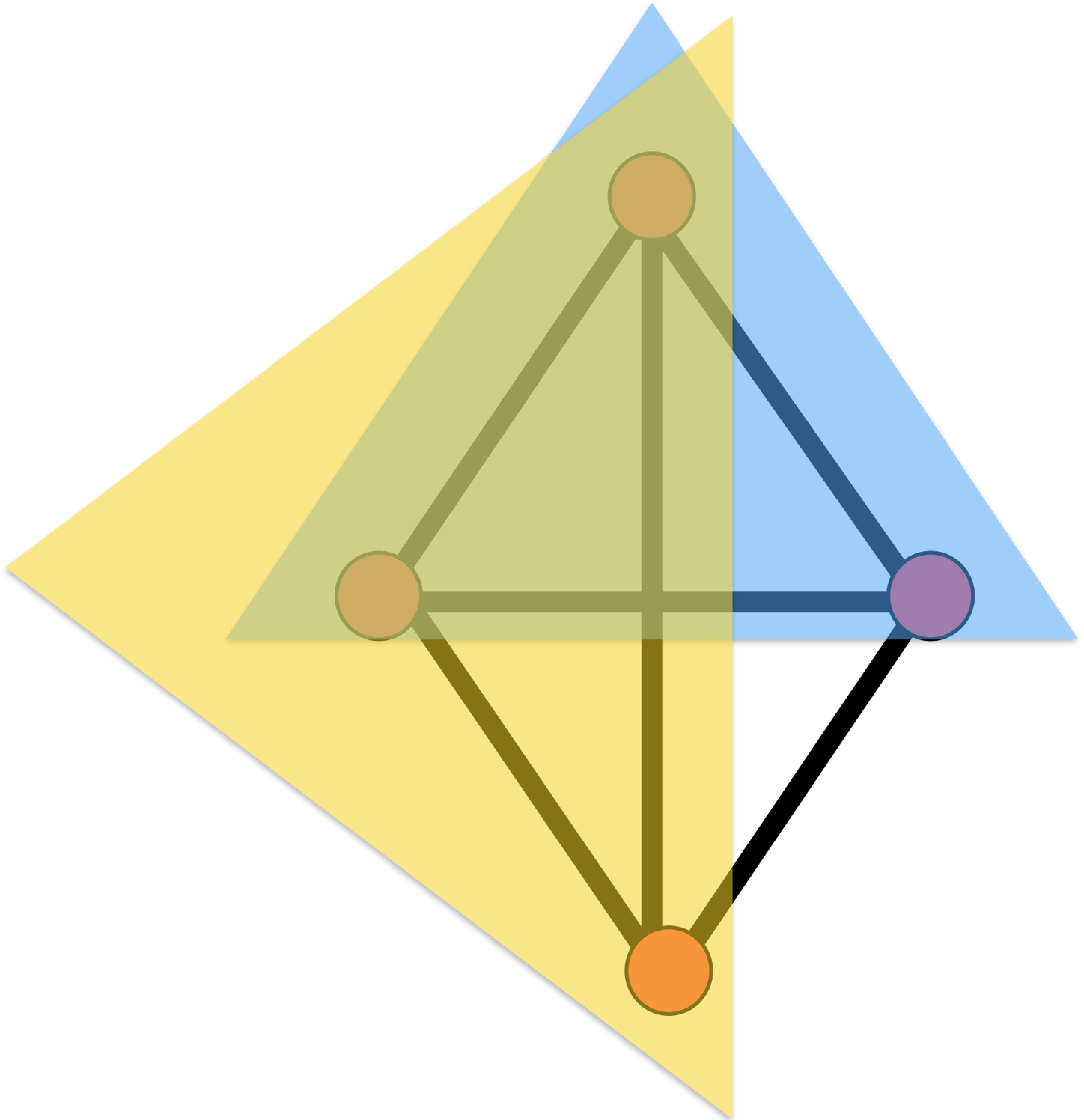


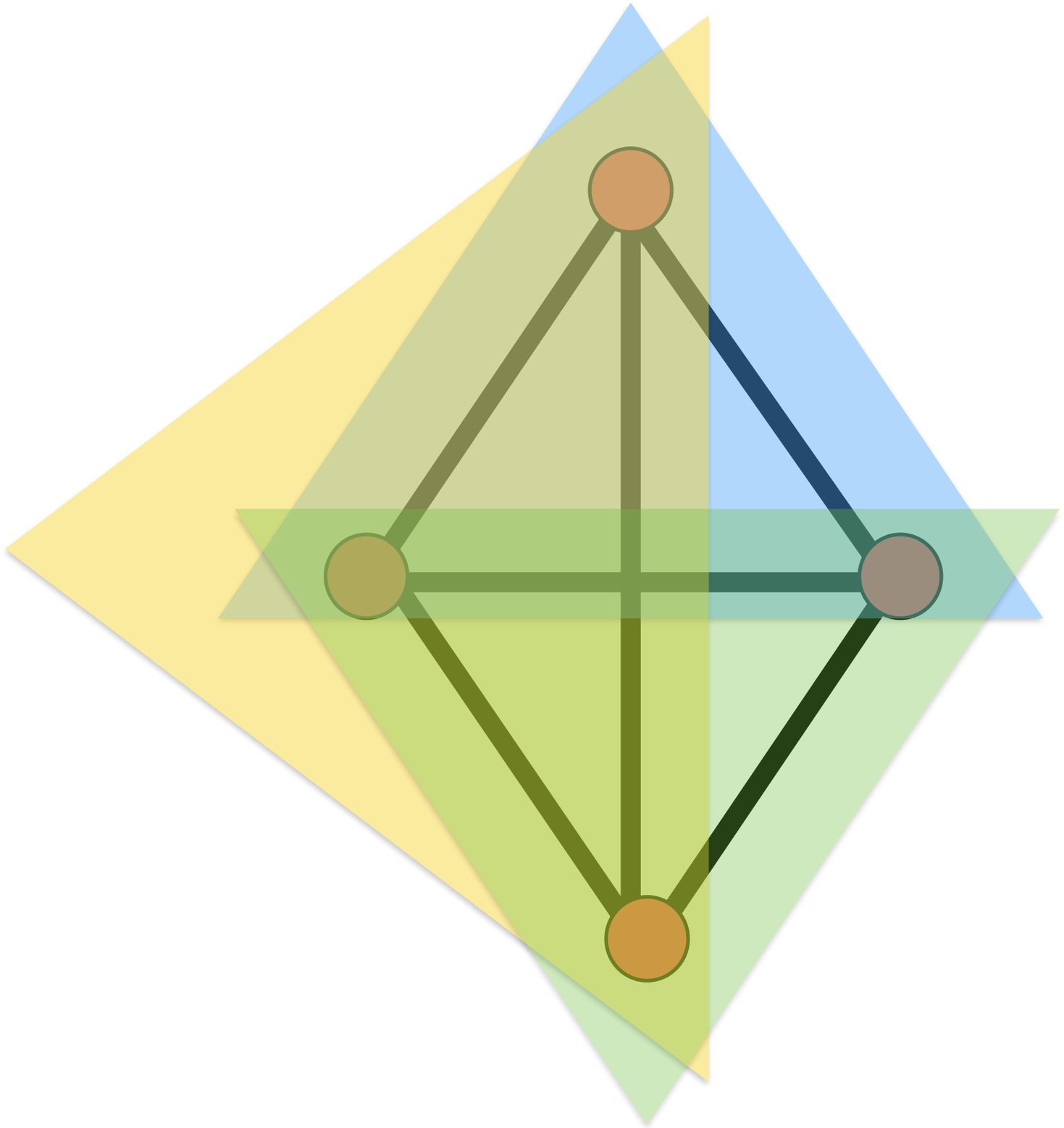






n=4, t=2, k=3





How can we find good/optimal covering designs?

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You have to be careful!

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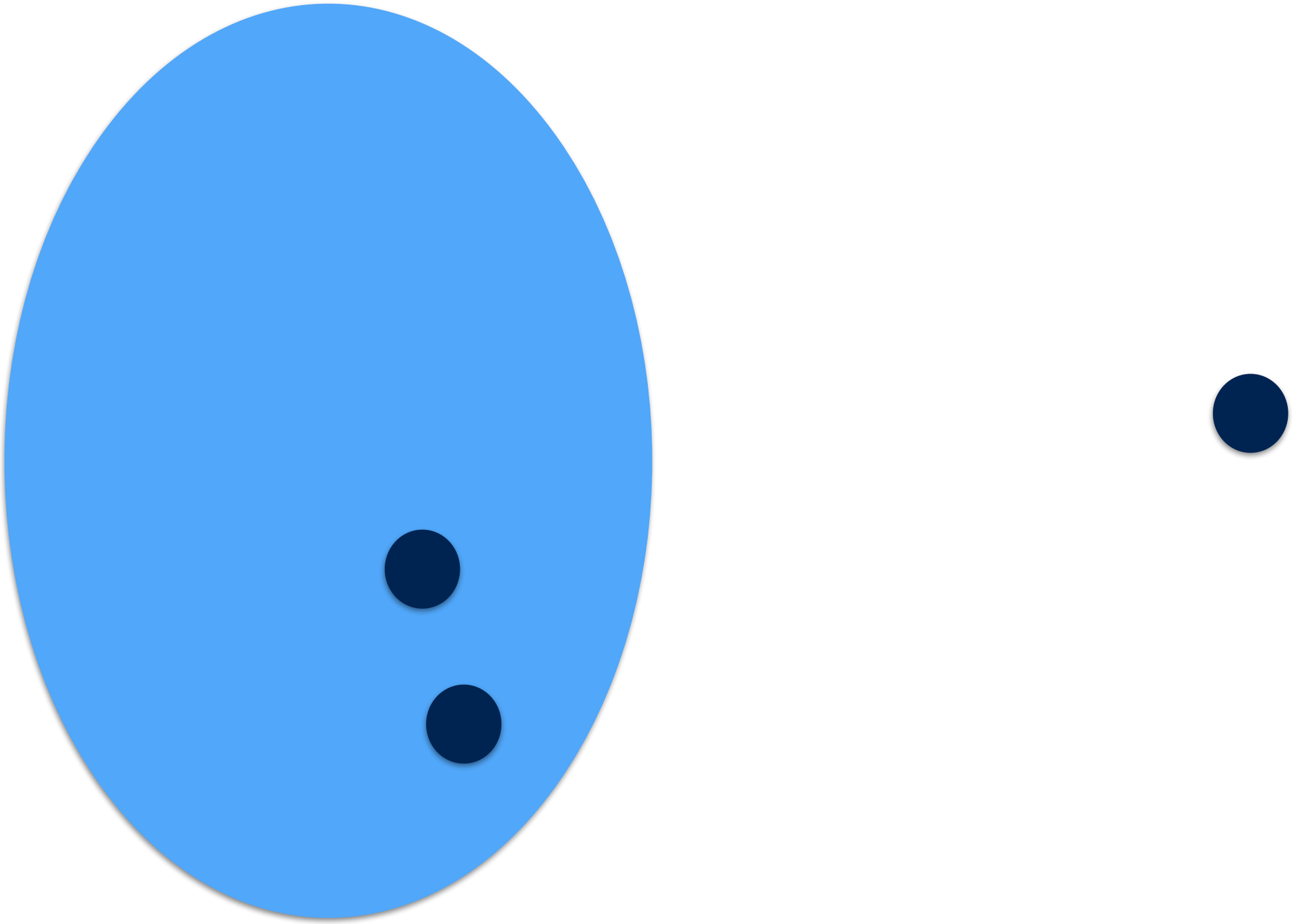
There are 11,084,874,829 optimal designs
with n=19, t=2, k=3

A seed with n-1 objects



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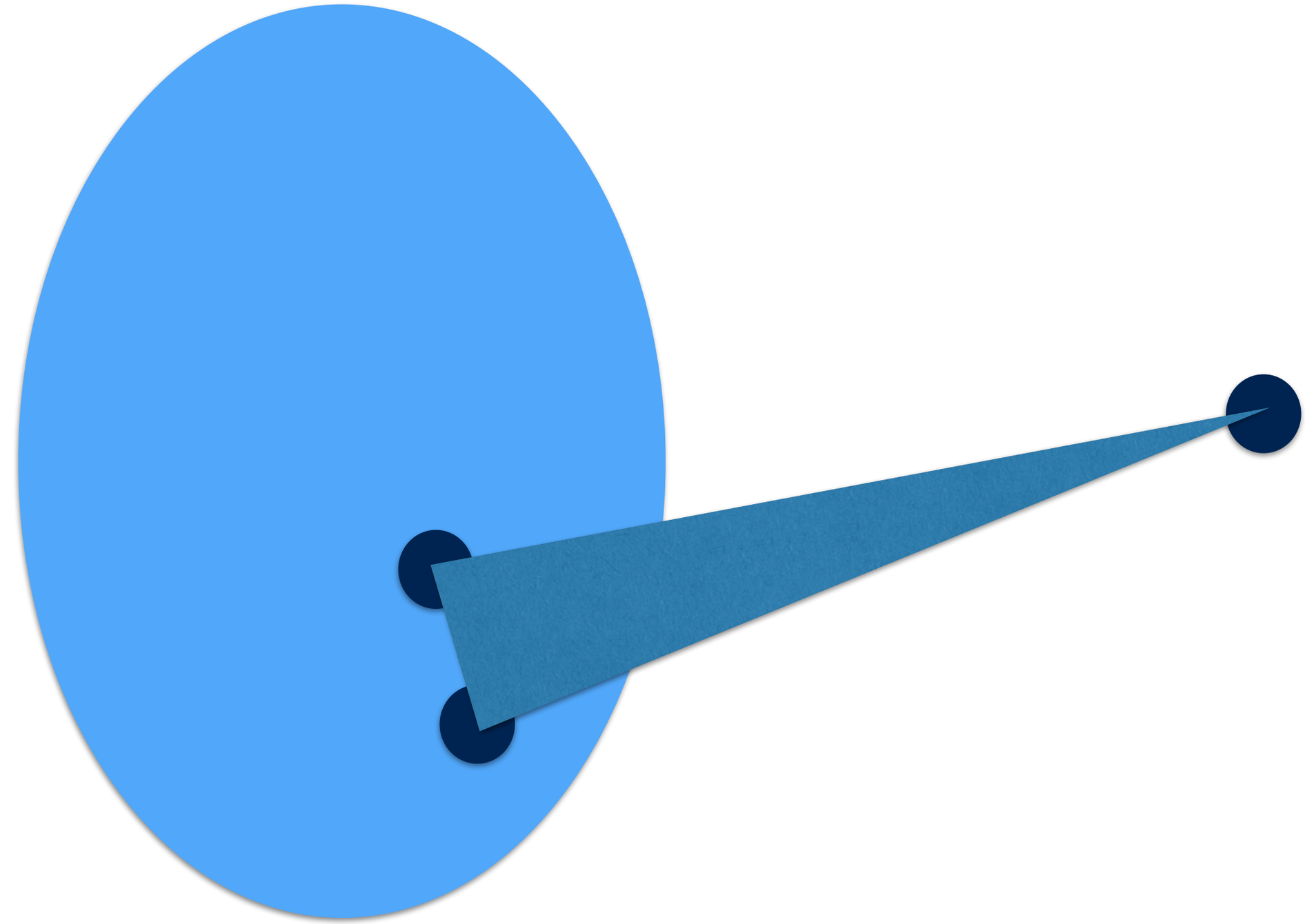

- For each seed, create a linear program
- Find all integer solutions to the linear programs
- Reduce the set of all solutions to nonequivalent ones
- Repeat the process for near-optimal solutions

- Using this process we have found all designs, with all parameters up to n=10 objects
- Up to n=12 we have the optimal designs for most parameters

N=10

k/t	8	7	6	5	4	3	2
8	, 1						
7		, 1	45, 20	20, 5	10, 2	6, 5	3, 1
6			, 1	50, 1	20, 1	10, 72	4, 1
5				, 1	51, 40	17, 218	6, 2
4					, 1	30, 1	9, 4
3						, 1	17, 58
2							, 1

N=11

k/t	9	8	7	6	5	4	3	2
9	, 1							
8		, 1	63, 40	29, 1	16, 38	9, 10	5, 1	3, 1
7			, 1	84, 3	34, 8	17, 136	8, 9	4, 3
6				, 1	>=98, >=1	32, 1	11, 1	6, 277
5					, 1	66, 1	20, 1	7, 2
4						, 1	47, 121500	11, 2
3							, 1	19, 2

- Using this process we have found all designs, with all parameters up to n=10 objects
- Up to n=12 we have the optimal designs for most parameters
- Some parameters complete up to n=20
- Some parameter combinations up to n=26

- All resulting designs can be downloaded freely
- About 4-5000 downloads per year since 2009
- Users include private individuals, university researchers, private companies, government organisations, European Medicines Agency

We also have a similar archive for double-covering designs.

In order to cover everything twice one can of course use two covering designs, but this is often very inefficient

We investigate other extremal problems as well.

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- Asymptotics for designs, using semidefinite programming
- Snark graphs. Gave counterexamples to several mathematical conjectures
- Enumeration problems with connection to matrix permanents
- Voting systems which minimize the risk for inconsistencies